Mathematics of the Mass Increase

HOME: The Physics of Bruce Harvey

In this revision of this section (11/06/2009) we introduce a correction which enables our unified theory to correctly account for the slowing of both mechanical and atomic clocks by motion through the background.

The effects of motion through the background on the electron and the U and D quarks are far from simple to understand. The result however is very simple in the case of high energy elementary charged particles. The laboratory approximates to the stationary system from which a force is exerted to accelerate the particle to near light speed. The particle resists this as if it had a mass of $\gamma^3 m_0$. When the integrals are performed, we find that the particle has acquired a kinetic energy of $(\gamma - 1) m_0 c^2$ which means that its total energy has increased to $\gamma m_0 c^2$. Such simple results can be derived through any number of erroneous theories!

Our unified theory is not allowed any loose ends. Everything must be explained by simple causal processes which nature can enact. Our observations reveal that the mass of high energy particles is increased, time dependant processes run slow and all attempts to detect the motion of the laboratory through the background fail. While the relativity based on the ideas of Lorentz and Poincaré explained these as real physical effects, Einstein's theory explained them away as artefacts of observation. Much was made of the twins paradox which sought to show the foolishness of Einstein's theory, but was turned on its head and presented as a mystery which only the most intelligent minds could grasp. With the advent of the GPS system, the exact effect on clocks was at once revealed to be real. However, the atomic clocks actually measure atomic energy levels because they count a beat frequency dependent on the relationship $E = h\nu$ determining the frequency of a photon from its energy content. So while we are taught that mass increases, we observe that energy levels appear to have been reduced: a combination which would seem to contradict the equivalence of mass and energy.

In our unified theory, the electric fields of all elementary charged particles coexisting in space form a background against which the motion of electric flux generates magnetic intensity and the motion of magnetic flux generates electric intensity. This background takes the place of the æther in earlier theories enabling both photons and radio waves to exist and travel through it at constant speed[1]. The motion through the background of the electric flux of each individual elementary charged particle generates a magnetic field containing its kinetic energy.

To make sense of nature, we must distinguish between actions which take place within the background and those which take place within the moving system. When we consider the acceleration of high energy charged particles, the effect of the motion of their electric fields through the background generates magnetic intensity. This action takes place within the background as does the action of the force doing work by acting though a distance. However, the resulting formation of magnetic flux, the motor action of turning mechanical energy into magnetic energy stored within the flux and the movement of the flux from the surface out into the field are actions which take place in the moving system. The Lorentz transforms are too crude a tool to cope with this distinction.

The Lorentz transforms relate to the situation where two observers are in relative motion and both record the position and time of events. Under the right set of conditions imposed on the way they measure position and time, the transforms allow them to convert the positions and times of an event from one one observer's

1 The concept of the constant speed of light is very complicated because the speed of light is affected by both gravitational potential and motion through the background in the same ways that the speed of sound is affected by temperature and wind. However, both gravitational potential and motion through the background affect our rulers and clocks such that we always measure the speed of light to be the same.

measurements to those of the other. We relate the observations we make in our moving system with those of an imaginary observer who is stationary in the background.

The real situation is that our inability to detect the earth's motion through the background reveals the fact that our rulers have contracted in the direction of motion and our clocks slowed. The situation is very similar to an increase in temperature causing rulers to expand and clocks to slow. Engineers have been to great length to perfect rulers and clocks which are unaffected by temperature and we can use these to measure the effects of temperature on ordinary rulers and clocks. To understand the effects of motion through the background, we must imagine that we have some god-given set of rulers, clocks and other measuring instruments which are unaffected by motion. What interests us is the relationship between measurements made within the moving system and those which would be obtained if we had god-given measuring instruments. We will distinguish this from the LTs of relativity by calling it a "Lorentz mapping" and writing with the symbolism of a mapping: $measured \rightarrow actual$.

If we try to observe the mass of a moving object from within the stationary system, which is effectively what we are doing in particle physics, then we do indeed find that $m_t = \gamma\, m_0$ and $m_l = \gamma^3\, m_0$. If on the other hand we consider the increase in mass which we would observe from within the moving system with a set of god-given rulers and clocks, we would find that $m_t \rightarrow \gamma^2\, m_0$ and $m_l \rightarrow \gamma^4\, m_0$. This makes good sense when we consider the mappings for Newton's second law for acceleration in the direction of the motion of the moving system:

$$l \rightarrow \frac{l}{\gamma} \qquad t \rightarrow \gamma\, t \qquad v \rightarrow \frac{v}{\gamma^2} \qquad a \rightarrow \frac{a}{\gamma^3} \qquad F \rightarrow \gamma\, F$$

If for example $\gamma = \frac{5}{4}$ and we measure velocity over 1000m timed at 40 seconds, then $1000 \rightarrow 800$ and $40 \rightarrow 50$ so the velocity $\frac{1000}{40} \rightarrow \frac{800}{50}$ which is $25 \rightarrow 16$. Since all forces are exerted through the electric fields $\vec{E} = \nabla\phi$ of elementary charged particles, they are directly affected by the contraction and increased by a factor γ.

$$F = m\, a \qquad \rightarrow \qquad \gamma F = \left(\gamma^4 m\right)\left(\frac{a}{\gamma^3}\right)$$

The Lorentz transform would give $F = \left(\gamma^3 m\right)\left(\frac{a}{\gamma^3}\right)$ because the forces in the direction of motion are said to be equal as indeed they are from the point of view of the observer in the stationary system. This paradox becomes a little easier to understand if we imagine the force to be exerted by pulling on a rope. If the observer in the stationary system pulls in the rope, then the rope is part of the moving system and consequently contracted. On the other hand, if the observer in the moving system pulls in the rope, then the rope is part of the stationary system and is uncontracted. This results in a velocity ratio and consequently a mechanical advantage.

When we come to consider a hydrogen atom as the moving system, we find that angular momentum is conserved.

Our previous proof that the energy content of the electric field of the electron is invariant was in error because we followed an original mistake of Lorentz. Using Cartesian Co-ordinates, we had correctly assumed that the contraction increases the y and z components of the electric flux density $\vec{D}$ and increases the x component of the electric field strength $\vec{E}$ which is equal to the gradient of the potential $\nabla\phi$. We were in error in accepting Lorentz's conclusion that the reduced capacity of the volume element $d\tau$ results in a decrease in the result of the integral. The volume element $\delta\tau$ remains the same volume element containing the same chunk of flux and what were co-ordinates in Euclidean space describing its position and size have now become parameters. This turns the integral into a parametric integral summing all such volume elements.

$$\vec{D}' = \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix} \qquad \vec{E}' = \nabla\phi' = \begin{pmatrix} \gamma \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix}$$

And $$\mathscr{E}_e' = \int \frac{1}{2} \vec{D}' \cdot \vec{E}' \, d\tau = \int \frac{1}{2} \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix} \cdot \begin{pmatrix} \gamma \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix} d\tau = \int \frac{1}{2} \gamma \vec{D} \cdot \vec{E} \, d\tau$$

This becomes

$$\mathscr{E} = \int \vec{D} \cdot \nabla\phi \, d\tau = \int_0^{2\pi} \int_0^{\pi} \int_a^{\infty} \left(\frac{q}{4\pi r^2} \right)^2 \begin{pmatrix} \cos\theta \\ \gamma \sin\theta \cos\phi \\ \gamma \sin\theta \sin\phi \end{pmatrix} \cdot \begin{pmatrix} \gamma \cos\theta \\ \sin\theta \cos\phi \\ \sin\theta \sin\phi \end{pmatrix} r^2 \sin\theta \, dr \, d\theta \, d\varphi = \gamma \frac{q^2}{8\pi \varepsilon_0 a}$$

We now assert that the energy content of the electric field is invariant because there is no mechanism for altering it. So we conclude that the integration describes the action of the contraction within the electron's flux. Since charge and permittivity are invariant, the only possible explanation is that the parameter a must have increased by a factor γ giving:

$$\mathscr{E} = \int \vec{D} \cdot \nabla\phi \, d\tau = \gamma \frac{q^2}{8\pi \varepsilon_0 (\gamma a)} = \frac{q^2}{8\pi \varepsilon_0 a}$$

Lorentz performed his integrations using the auxiliary co-ordinates. The raw results were $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$. He then argued that as the real co-ordinates were contracted, he needed to divide by a factor of γ to give the final result $m_l = \gamma^3 m_0$ and $m_t = \gamma m_0$. This led him to reject his own theory in favour of Abraham's theory which agreed more closely with the experimental data. For our purposes here, we would prefer not to have to divide by γ leaving the result as $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$ because this is consistent with the effect on time dependent processes.

What Lorentz failed to realise was that his auxiliary co-ordinates are in fact parameters and the integrals he performed were parametric integrals. It is possible to change the parametric integral into an integral over Euclidean space only to get the same result. The mathematics looks similar to that for the energy content of the electric field. Deceptively similarly: which is why Lorentz made the mistake of dividing by γ to take into account the reduced volume of the volume element. We now believe this to be an error. The raw results must still be divided by γ but now for the reason that the contraction has increased the energy density of the electric flux disturbing the equilibrium which determines the radius of the electron. The parameter a has increased by a factor γ.

At slow speeds, the energy content of the magnetic field of an electron is:

$$\mathscr{E} = \int \mu_0 \left(\vec{u} \wedge \vec{D} \right)^2 d\tau = \int_0^{2\pi} \int_0^{\pi} \int_a^{\infty} \frac{\mu_0 q^2 u^2 \sin^2\theta}{32\pi^2 r^4} r^2 \sin\theta \, dr \, d\theta \, d\varphi = \frac{\mu_0 q^2 u^2}{12\pi a}$$

As the speed increases and the Lorentz contraction becomes significant. It increases the energy density of the magnetic field by a factor γ and also affects the values of θ, $d\theta$, r and dr as measured against Euclidean space.

$$\theta \to \tan^{-1}(\gamma \tan\theta) \qquad d\theta \to \gamma \frac{1 + \tan^2\theta}{1 + \gamma^2 \tan^2\theta} d\theta \qquad \sin\theta \to \gamma \frac{\tan\theta}{\sqrt{1 + \gamma^2 \tan^2\theta}}$$

$$r \;\to\; r\sqrt{\frac{cos^2\theta}{\gamma^2} + (1 - cos^2\theta)} \qquad\qquad dr \;\to\; \sqrt{\frac{cos^2\theta}{\gamma^2} + (1 - cos^2\theta)}\; dr$$

The limit a becomes $\frac{a}{\sqrt{\frac{cos^2\theta}{\gamma^2} + (1 - cos^2\theta)}}$ as can be seen in the example:

$$\int_a^\infty \frac{1}{r^2}\, dr \;=\; \frac{1}{a} \qquad \to \qquad \int_{\frac{a}{k}}^\infty \frac{1}{(k\, r)^2}\, k\, dr \;=\; \frac{1}{a}$$

When these new values are substituted into the integral, the function tan^{-1} is only well defined from $-\pi / 2$ to $\pi / 2$, so we need to integrate over θ for half the interval and double the result. We must also change the order of integration because r is now a function of θ. The integral becomes:

$$2 \int_0^{2\pi} \int_{\frac{a}{\sqrt{\frac{cos^2\theta}{\gamma^2} + (1 - cos^2\theta)}}}^{\infty} \int_0^{\frac{\pi}{2}} \gamma^2 \frac{\mu_0\, q^2\, u^2 \left(\gamma \frac{tan\theta}{\sqrt{1 + \gamma^2\, tan^2\theta}}\right)^3}{32\pi^2 \left(\frac{cos^2\theta}{\gamma^2} + \left(1 - cos^2\theta\right)\right) r^2} \sqrt{\frac{cos^2\theta}{\gamma^2} + (1 - cos^2\theta)}\;\; \gamma \frac{1 + tan^2\theta}{1 + \gamma^2\, tan^2\theta}\, d\theta\, dr\, d\varphi$$

Fortunately MathCad makes light work of this to get:

$$\mathcal{E} \;=\; \gamma^2 \frac{\mu_0\, q^2\, u^2}{12\pi\, a}$$

Which could be found much more simply from:

$$\int_0^{2\pi} \int_0^{\pi} \int_a^{\infty} \gamma^2 \frac{\mu_0\, q^2\, u^2\, sin^2\theta}{32\pi^2\, r^4}\, r^2\, sin\,\theta\; dr\; d\theta\; d\varphi \;=\; \gamma^2 \frac{\mu_0\, q^2\, u^2}{12\pi\, a}$$

Since while the values of θ, $d\theta$, r and dr have changed, θ, φ and r are still parameters which describe a way of dividing space into volume elements and give the energy density within the volume element. So what we have here is an example of parametric integration. If we had simply performed it, the reader would have thought it a fudge.

These actions through which the motion of the electric field generates magnetic field and feedback effects cause the Lorentz contraction all take place within the magnetic flux. The equilibrium which determines the size of the electron is a separate physical action, so as a increases in magnitude to maintain the energy content of the electric field, the algebra of the integrations is unaffected delivering the result $\gamma^2 \frac{\mu_0\, q^2\, u^2}{12\pi\, a}$, but now we must take into account the increase in the magnitude of a giving:

$$\mathcal{E} \;=\; \gamma^2 \frac{\mu_0\, q^2\, u^2}{12\pi\, (\gamma\, a)} \;=\; \mathcal{E} \;=\; \gamma \frac{\mu_0\, q^2\, u^2}{12\pi\, a}$$

If we are experimenting with high energy particles, the laboratory is in effect the stationary system and we see the relativistic mass increasing by a factor of γ. We find that the particle resists linear acceleration as if its mass were increased by a factor γ^3 because work must also be done to increase the value of γ.

On the other hand, when we observe an atomic clock, we are now part of the moving system. All our instruments are affected by the contraction. We are quite unaware of the effects of the contraction and its effects on mass and energy. Given that we learn through communicating with the outside world that our clocks have slowed, we examine the equations governing oscillations and mechanical clocks and discover that mass seems to have increased by a factor of γ^2. But then, turning to quantum theory, we discover that the energy of a photon $h\,\nu$ must be reduced because h is invariant. This implies a reduction in the energy levels within the atom. In our unified theory, the orbital frequency of an electron is determined by a combination of orbital mechanics, electromagnetism, so we will return to the subject in our section "Relativistic Quantum

Theory".

We diverge considerably from Lorentz's simple derivation which does not give an adequate explanation of centrifugal force. Lorentz deduced the relationship between transverse and longitudinal mass. He derived the longitudinal mass from an integration of the total energy content of the magnetic field. This is very much simpler than the approach we have followed. In order to unmask the process by which centrifugal force is generated, we have to consider the rate of change in energy content of the magnetic field. Now there is no net change, only a rotation of the magnetic field as the direction of motion changes (the flux being orthogonal to the direction of motion). The only way to obtain the correct result is to assume that changes in the magnetic field must be accommodated by the movement of energy parallel to the electric field. Nature does not allow the electric field to rotate, so we may construct a volume element outwards from a surface element parallel to $\vec{D}$ and consider the energy content of the magnetic field within it. This gives us a rate of change of energy within the volume element which we equate to a force on the surface element. The centrifugal force exists as the sum of these forces over the surface elements.

Before we can proceed with the derivation, we need to quote two identities of which the second one is not obvious and from the algebraic perspective, it is very unusual:

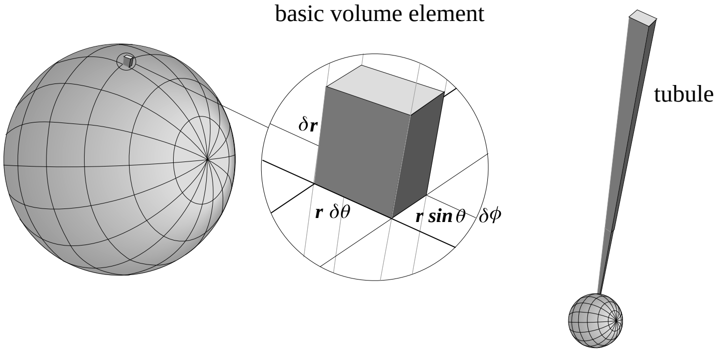
$$\frac{d}{dt}\gamma^n = \frac{n\,v}{c^2}\gamma^{n+2}\frac{dv}{dt} \qquad \text{and} \qquad \frac{v^2}{c^2}\gamma^2 + 1 = \gamma^2$$

We will also need to understand that for functions containing scalars and vectors, the normal rules for differentiating compound functions apply so long as the type and order of multiplication are preserved. Finally we use a technique in which the quadruple scalar product $\vec{A} \wedge \vec{B} \cdot \vec{C} \wedge \vec{D}$ may be treated as a triple scalar product and subjected to cyclic rotation, for example:

$$\vec{A} \wedge \vec{B} \cdot \vec{C} \wedge \vec{D} = \vec{A} \wedge \vec{B} \cdot \left(\vec{C} \wedge \vec{D}\right) = \vec{B} \wedge \left(\vec{C} \wedge \vec{D}\right) \cdot \vec{A}$$

The following derivation is simple when one knows how to do it, but the fact that all of the above need to be used make its discovery almost impossible.

According to classical theory, the moving charge is surrounded by a magnetic field $\vec{B} = \frac{\mu_0\,q}{4\,\pi\,r^2}\vec{v} \wedge \hat{r}$. The Lorentz contraction increases the electric flux density $\vec{D} = \frac{q}{4\,\pi\,r^2}\hat{r}$ by a factor of $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ increasing the magnetic flux density to $\vec{B} = \frac{\gamma\,\mu_0\,q}{4\,\pi\,r^2}\vec{v} \wedge \hat{r}$. The magnetic field has an energy density $\frac{1}{2\,\mu_0}B^2 = \frac{\gamma^2\,\mu_0\,q^2}{32\,\pi^2\,r^4}\,(\vec{v} \wedge \hat{r})^2$. We consider a surface element of area $\delta A = r^2\,\delta\omega = r^2\,sin\,\theta\,\delta\theta\,\delta\phi$ and the conical volume element which can be constructed outwards from the surface element everywhere parallel to the electric field. Such volume elements will herein after be referred to as "tubules" since they are constructed according to the rule devised by Faraday to define what latter became known as "Faraday tubes".



We work in Lorentz's auxiliary co-ordinates which move with the electron and suffer a contraction in length. From the discussion above, we know that they have become parameters describing the geometry of the field and giving the energy density when account is taken of the effect of the contraction on energy density. The basic volume element of the tubule is $\delta\tau = r^2\,\delta\omega\,\delta r$ and the energy content of the magnetic field within the tubule is:

$$\delta\mathscr{E}_m = \frac{\gamma^2\mu_0 q^2}{32\pi^2}(\vec{v}\wedge\hat{r})^2\int_{r_0}^{\infty}\frac{1}{r^4}\,r^2\delta\omega\,dr = \frac{\gamma^2\mu_0 q^2}{32\pi^2 r_0}(\vec{v}\wedge\hat{r})^2\,\delta\omega$$

We can now differentiate this with respect to time to find the rate of change of energy content of the magnetic field within the tubule. We must remember that γ is a function of velocity. Everything which remains constant with time can be left outside the differentiation.

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{d}{dt}\left(\frac{\gamma^2\mu_0 q^2}{32\pi^2 r_0}(\vec{v}\wedge\hat{r})^2\,\delta\omega\right) = \frac{\mu_0 q^2}{32\pi^2 r_0}\frac{d}{dt}\left(\gamma^2(\vec{v}\wedge\hat{r})^2\right)\delta\omega$$

$$\frac{d}{dt}\gamma^2(\vec{v}\wedge\hat{r})^2 = \frac{d}{dt}\left(\gamma^2\right)(\vec{v}\wedge\hat{r})^2 + \gamma^2\frac{d}{dt}\left((\vec{v}\wedge\hat{r})^2\right)$$

$$= \frac{2v}{c^2}\gamma^4\frac{dv}{dt}(\vec{v}\wedge\hat{r})^2 + 2\gamma^2(\vec{v}\wedge\hat{r})\cdot\left(\frac{d\vec{v}}{dt}\wedge\hat{r}\right)$$

We write $\frac{d\vec{v}}{dt} = \vec{a}$ as $a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and note that at this instant $\hat{v} = \hat{i}$ and $\vec{v} = v\,\hat{i}$. Then $\frac{dv}{dt} = a_x$ since it measures the rate of change in magnitude in $\vec{v}$.

$$\frac{d}{dt}\gamma^2(\vec{v}\wedge\hat{r})^2 = 2\gamma^2\left(a_x\frac{v}{c^2}\gamma^2\vec{v}\wedge\hat{r} + \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)\wedge\hat{r}\right)\cdot\left(\vec{v}\wedge\hat{r}\right)$$

The next step is to separate the magnitude v and direction $\hat{i}$ of $\vec{v}$ and rearrange $a_x\frac{v}{c^2}\gamma^2\vec{v} \rightarrow \frac{v^2}{c^2}\gamma^2 a_x\hat{i}$. Then collecting the two terms containing a_x and using the γ^2 identity:

$$\frac{d}{dt}\gamma^2(\vec{v}\wedge\hat{r})^2 = 2\gamma^2\left(\left(\gamma^2 a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)\wedge\hat{r}\right)\cdot\left(\vec{v}\wedge\hat{r}\right)$$

Writing $\vec{a}_\gamma = \gamma^2 a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and turning the quadruple scalar product into a triple scalar product:

$$\frac{d}{dt}\gamma^2\left(\vec{v}\wedge\hat{r}\right)^2 = 2\gamma^2\;\vec{v}\wedge\hat{r}\cdot\left(\vec{a}_\gamma\wedge\hat{r}\right)$$

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{d}{dt}\left(\frac{\gamma^2\mu_0 q^2}{32\pi^2 r_0}(\vec{v}\wedge\hat{r})^2\,\delta\omega\right) = \frac{\mu_0 q^2}{32\pi^2 r_0}\;2\gamma^2\,\hat{r}\wedge\left(\vec{a}_\gamma\wedge\hat{r}\right)\cdot\vec{v}\,\delta\omega$$

We are now in a position to equate the change in energy with the rate of work done by a force moving with velocity $\vec{v}$ in time δt.

$$\delta\vec{F}\cdot\vec{v}\delta t = \frac{d}{dt}\delta\mathscr{E}_m\,\delta t$$

$$\delta\vec{F}\cdot\vec{v} = \frac{\mu_0 q^2}{32\pi^2 r_0}\;2\gamma^2\delta\omega\;\hat{r}\wedge\left(\vec{a}_\gamma\wedge\hat{r}\right)\cdot\vec{v}$$

Note that although we chose to impose co-ordinates with the x axis along the direction of motion, this equation requires only that the origin is at the centre of the sphere. Although the dot product does not in general cancel, the fact that this is true for all $\vec{v}$ in this equation implies that:

$$\delta\vec{F} \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\;\gamma^2\;\hat{r}\,\wedge\,\left(\vec{a}_\gamma\,\wedge\,\hat{r}\right)\,\delta\omega$$

Let us remind ourselves that we have just found the force on the surface element of solid angle $\delta\omega$. We may now integrate over the area of the sphere to find the force required to produce the centripetal acceleration.

$$\vec{F} \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\gamma^2\;\int\;\hat{r}\,\wedge\,\left(\vec{a}_\gamma\,\wedge\,\hat{r}\right)\,d\omega$$

$$\vec{F} \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\,\gamma^2\int_0^{2\pi}\int_0^{\pi}\;\hat{r}\,\wedge\,\left(\vec{a}_\gamma\,\wedge\,\hat{r}\right)\,\sin\theta\;d\theta\;d\phi$$

This calculation is best done in Cartesian co-ordinates expanding the vector product, then integrating. The essentials of this have been captured from a Mathcad file.

$$\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\times\left[\begin{pmatrix}\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}\times\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\right] = \begin{pmatrix}-\sin(\theta)\cdot\cos(\phi)\cdot a_y\cdot\cos(\theta) - \sin(\theta)\cdot\sin(\phi)\cdot a_z\cdot\cos(\theta) + \gamma^2\cdot a_x - \gamma^2\cdot a_x\cdot\cos(\theta)^2\\ a_y - a_y\cdot\cos(\phi)^2 + a_y\cdot\cos(\theta)^2\cdot\cos(\phi)^2 - \sin(\phi)\cdot a_z\cdot\cos(\phi) + \sin(\phi)\cdot a_z\cdot\cos(\phi)\cdot\cos(\theta)^2 - \cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\cos(\phi)\\ a_z\cdot\cos(\theta)^2 - \cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\sin(\phi) - \cos(\phi)\cdot a_y\cdot\sin(\phi) + \cos(\phi)\cdot a_y\cdot\sin(\phi)\cdot\cos(\theta)^2 + \cos(\phi)^2\cdot a_z - \cos(\phi)^2\cdot a_z\cdot\cos(\theta)^2\end{pmatrix}$$

$$\int_0^{2\pi}\int_0^{\pi}\left(-\sin(\theta)\cdot\cos(\phi)\cdot a_y\cdot\cos(\theta) - \sin(\theta)\cdot\sin(\phi)\cdot a_z\cdot\cos(\theta) + \gamma^2\cdot a_x - \gamma^2\cdot a_x\cdot\cos(\theta)^2\right)\cdot\sin(\theta)\,d\theta\,d\phi \rightarrow \frac{8}{3}\cdot\pi\cdot\gamma^2\cdot a_x$$

$$\int_0^{2\pi}\int_0^{\pi}\left(a_y - a_y\cdot\cos(\phi)^2 + a_y\cdot\cos(\theta)^2\cdot\cos(\phi)^2 - \sin(\phi)\cdot a_z\cdot\cos(\phi) + \sin(\phi)\cdot a_z\cdot\cos(\phi)\cdot\cos(\theta)^2 - \cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\cos(\phi)\right)\cdot\sin(\theta)\,d\theta\,d\phi \rightarrow \frac{8}{3}\cdot a_y\cdot\pi$$

$$\int_0^{2\pi}\int_0^{\pi}\left(a_z\cdot\cos(\theta)^2 - \cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\sin(\phi) - \cos(\phi)\cdot a_y\cdot\sin(\phi) + \cos(\phi)\cdot a_y\cdot\sin(\phi)\cdot\cos(\theta)^2 + \cos(\phi)^2\cdot a_z - \cos(\phi)^2\cdot a_z\cdot\cos(\theta)^2\right)\cdot\sin(\theta)\,d\theta\,d\phi \rightarrow \frac{8}{3}\cdot a_z\cdot\pi$$

$$\int_0^{2\pi}\int_0^{\pi}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\times\left[\begin{pmatrix}\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}\times\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\right]\cdot\sin(\theta)\,d\theta\,d\phi = \begin{pmatrix}\frac{8}{3}\cdot\pi\cdot\gamma^2\cdot a_x\\ \frac{8}{3}\pi\cdot a_y\\ \frac{8}{3}\pi\cdot a_z\end{pmatrix}$$

[This may be viewed at up to 400% in Acrobat and will print legibly]

Writing the result of the integration as $\frac{8\pi}{3}\,\vec{a}_\gamma$, the force required to accelerate the electron is:

$$\vec{F} \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\,\gamma^2\,\frac{8\pi}{3}\,\vec{a}_\gamma \;\;=\;\; \frac{\mu_0\, q^2}{6\,\pi\, r_0}\,\gamma^2\;\vec{a}_\gamma$$

Defining a quantity $m_0 \;=\; \dfrac{\mu_0\, q^2}{6\,\pi\, r_0}$ we have:

$$\vec{F} \;=\; \gamma^2\, m_0\,\vec{a}_\gamma$$

This is a relativistic form of Newton's second law.

This does not appear to support the empirical result for longitudinal and transverse mass. There must be a simple explanation.

The explanation is that in accelerating high energy particles, we are applying a force from what is effectively the stationary system to effect an action in the moving system. Let us suppose that we exerting the force by means of an inextensible string. The acceleration is achieved by exerting a force as we wind in the string. The problem is that the string is Lorentz contracted as it enters our winch, but has returned to its original length when it lies coiled beside the winch. This gives us a mechanical advantage of γ. Now obviously, we cannot use a string and a winch, but must rely on an electric field. The effect of the contraction is to give the electric field a mechanical advantage so that we observe it to be:

$$\vec{F} = \gamma m_0 \vec{a}_\gamma$$

Transverse mass is observed when the charge passes through a magnetic field and it accelerated perpendicular to its motion by the so called *Bev* force. In our unified theory, the charge is surrounded by its own magnetic flux which moves with it and pushes apart the magnetic flux of the background magnetic field. The interaction is not a local interaction between the surface of the charge and the background magnetic field, but is an interaction between the magnetic intensity $\vec{H}$ fields over all space. The Lorentz contraction increases the magnetic intensity generated by the motion of the charges electric field. We have dealt with this in the section Force on a Charge in a Magnetic Field within the relativity sections. The net result is that the magnetic force on a charge moving at near light speed is also increased by a factor γ.

Therefore the relativistic form of Newton's Law as seen from the system in which the force is applied acting on an electron moving through it is given by:

$$\vec{F} = \gamma m_0 \vec{a}_\gamma$$