Electromagnetic mass

HOME: The Physics of Bruce Harvey

Introduction

The discovery of the electron by JJ Thompson led to a frenzy of speculation about the nature of matter. Some, including JJ himself, thought that everything could be explained by electromagnetism. The stumbling block was that matter was believed to have the fundamental property of mass. What was mass, why should it have the properties of inertia and gravitational attraction? It took some time before Lorentz developed the theory of electromagnetic mass.

According to the laws of electromagnetism a moving electron should generate a magnetic field:

$$\vec{B} = \frac{\mu_0 \, q}{4 \, \pi \, r^2} \vec{v} \wedge \hat{r}$$

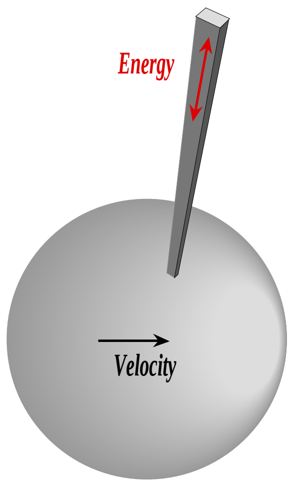
The realization that electrons are very small meant that the magnetic field could contain a significant amount of energy and possibly even its kinetic energy $\mathcal{E}_m = \frac{1}{2} m \, v^2$. If a is the radius of an electron, then:

$$\mathcal{E}_m = \int_{volume} \frac{1}{2\mu_0} B^2 = \frac{\mu_0 \, q^2}{12 \, \pi \, a} v^2$$

Lorentz postulated that the electron possessed two types of kinetic energy, one due to its mass and one due to its electromagnetic properties. He was then able to prove that the one due to its mass was zero and that the whole of its kinetic energy was stored in its magnetic field. This allowed Lorentz to calculate the radius of the electron:

$$\frac{1}{2} m \, v^2 = \frac{\mu_0 \, q^2}{12 \, \pi \, a} v^2 \qquad \Rightarrow \qquad a = \frac{\mu_0 \, q^2}{6 \, \pi \, m}$$

Which gives the value 1.879×10^{-15} m. This is smaller than the commonly quoted result of 2.8179×10^{-15} which comes from calculating the value of $\frac{e^2}{m \, c^2}$ in electrostatic units. The author considers this a good example of the mysticism of modern physics. A more correct analysis would equate the energy $m \, c^2$ with the energy $\frac{q^2}{8\pi \, \varepsilon_0 \, a}$ stored in the electric field of a charged spherical charge giving 1.409×10^{-15} m.

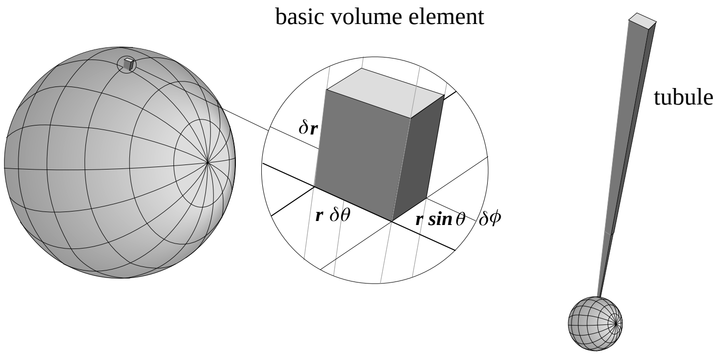


The original work by Lorentz and others simply equates changes in energy content with the work which done by a force. There is no attempt to identify or describe a mechanism by which the inertial force is generated. The author's first attempts identify such a mechanism were based on the laws of induction. In our unified theory, we identify the action of generating the inertial force as the fundamental process and derive the laws of induction from it. It is very difficult to explain why an electron should resist centripetal acceleration because its kinetic energy remains constant. The only action is the rotation of the field. We must assume that the kinetic energy stored in the magnetic field surrounding the electron can only move parallel to the electric flux. As the field rotates, energy has to be adsorbed in two quadrants and generated in the other two. We assume that a basic motor action takes place within the surface of the electron changing mechanical energy into magnetic energy and vice versa. We calculate the rate of change of the energy content of the magnetic field within a conic element of volume extending outwards from an element of area of the electron's surface. Equating this to a force and integrating over the whole surface gives the inertial force.

In itself, Lorentz's theory is unremarkable, but its triumph came from the fact that when he included the effect of the Lorentz contraction, it was able to explain why electrons moving at near light speed showed an increase in mass. This was one of the main components of the original relativity theory of Lorentz and Poincaré. However, the rising prominence of Einstein following the publication of his General Theory of Relativity led to Lorentz's work being lost in the mists of time even before the discovery of the neutron put an end to the theory of electromagnetic mass. Only with the development the three quark theory of nucleons in 1964 has it been possible to revise the theory.

Inertial force derivation (non-relativistic)

According to classical theory, the moving charge is surrounded by a magnetic field $\vec{B} = \frac{\mu_0 q}{4 \pi r^2} \vec{v} \wedge \hat{r}$. The magnetic field has an energy density $\frac{1}{2 \mu_0} B^2 = \frac{\mu_0 q^2}{32 \pi^2 r^4} (\vec{v} \wedge \hat{r})^2$. We consider a surface element of area $\delta A = r^2 \delta\omega = r^2 \sin\theta \, \delta\theta \, \delta\phi$ and the conical volume element which can be constructed outwards from the surface element everywhere parallel to the electric field. We will call these volume elements "tubules" since they are constructed according to the rule devised by Faraday to define what latter became known as "Faraday tubes".



The basic volume element of the tubule is $\delta\tau = r^2 \delta\omega \, \delta r$ and the energy content of the magnetic field within the tubule is:

$$\delta\mathscr{E}_m = \frac{\mu_0 q^2}{32 \pi^2} (\vec{v} \wedge \hat{r})^2 \int_{r_0}^{\infty} \frac{1}{r^4} r^2 \delta\omega \, dr = \frac{\mu_0 q^2}{32 \pi^2 r_0} (\vec{v} \wedge \hat{r})^2 \delta\omega$$

We can now differentiate this with respect to time to find the rate of change of energy content of the magnetic field within the tubule. Everything which remains constant with time can be left outside the differentiation.

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{d}{dt}\left(\frac{\mu_0 q^2}{32 \pi^2 r_0} (\vec{v} \wedge \hat{r})^2 \delta\omega \right) = \frac{\mu_0 q^2}{32 \pi^2 r_0} \frac{d}{dt}\left((\vec{v} \wedge \hat{r})^2 \right) \delta\omega$$

The rules of differentiation apply to a vector expression so long as the order and type of the vector operations is preserved. The unit vector $\hat{r}$ does not change with time and is not differentiated.

$$\frac{d}{dt} (\vec{v} \wedge \hat{r})^2 = 2 \left(\vec{v} \wedge \hat{r} \right) \cdot \left(\frac{d\vec{v}}{dt} \wedge \hat{r} \right)$$

We write the acceleration $\frac{d\vec{v}}{dt}$ as $\vec{a}$ and turn the quadruple scalar product of the form $\vec{A} \wedge \vec{B} \cdot \vec{C} \wedge \vec{D}$ into a triple scalar product of the form $\vec{A} \wedge \vec{B} \cdot \left(\vec{C} \wedge \vec{D}\right)$.

$$\frac{d}{dt}(\vec{v} \wedge \hat{r})^2 = 2\ \vec{v} \wedge \hat{r} \cdot (\vec{a} \wedge \hat{r})$$

The triple scalar product can be subjected to a cyclic rotation and the rate of change of the energy content of the tubule becomes:

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{\mu_0\, q^2}{16\, \pi^2\, r_0}\ \hat{r} \wedge (\vec{a} \wedge \hat{r}) \cdot \vec{v}\, \delta\omega$$

We are now in a position to equate the change in energy with the rate of work done by a force moving with velocity $\vec{v}$ in time δt.

$$\delta\vec{F} \cdot \vec{v}\delta t = \frac{d}{dt}\delta\mathscr{E}_m\, \delta t \qquad \Rightarrow \qquad \delta\vec{F} \cdot \vec{v} = \frac{\mu_0\, q^2}{16\, \pi^2\, r_0}\ \delta\omega\ \hat{r} \wedge (\vec{a} \wedge \hat{r}) \cdot \vec{v}$$

The normal rule is that the dot product does not cancel.

$$\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C} \quad \text{does not imply that} \quad \vec{B} = \vec{C}$$

However if $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$ is true for any two nonparallel $\vec{A}$ then $\vec{B} = \vec{C}$

So, since in this case, there is no restriction placed on the direction of the acceleration we may cancel giving:

$$\delta\vec{F} = \frac{\mu_0\, q^2}{16\, \pi^2\, r_0}\ \gamma^2\ \hat{r} \wedge (\vec{a} \wedge \hat{r})\ \delta\omega$$

This cancellation by the velocity $\vec{v}$ is central to our understanding of relative motion both in Newtonian mechanics and in relativity. Velocity is fundamentally a relative measurement and the physical circumstances determine how it should be measured. A pilot needs to know both the speed of the plane through the air and over the ground. It might seem to us as we drive up the motorway at 70 mph that this is an absolute speed and it certainly determines the kinetic energy which must be dissipated if we hit the back of a stationary lorry. However, our theory of electromagnetic mass can say no more than that there is a background of some form against which the motion of the electron's electric field generates the magnetic field containing its kinetic energy. The $\vec{v}$ of our equations is an absolute velocity, but we have no knowledge of the nature or locus of this background which Maxwell referred to as the "luminiferous æther". Fortunately, the accelerating force $\vec{F}$ acts against the background doing work at a rate of $\vec{F} \cdot \vec{v}$. Cancelling allows us to loose this inconvenient unknown velocity.

This $\delta\vec{F}$ is the force on the surface element of solid angle $\delta\omega$. We may now integrate over the surface area of the sphere to find the force required to produce the acceleration. Since the direction of $\vec{a}$ has not been specified, it consists of the two components of linear and centripetal acceleration.

$$\begin{aligned}\vec{F} &= \frac{\mu_0\, q^2}{16\, \pi^2\, r_0} \int \hat{r} \wedge (\vec{a} \wedge \hat{r})\ d\omega \\ &= \frac{\mu_0\, q^2}{16\, \pi^2\, r_0} \int_0^{2\pi} \int_0^{\pi} \hat{r} \wedge (\vec{a} \wedge \hat{r})\ sin\,\theta\ d\theta\ d\phi\end{aligned}$$

This integral is by no means simple to evaluate. The vector product $\hat{r} \wedge (\vec{a} \wedge \hat{r})$ must be evaluated in Cartesian co-ordinates and three integrals performed.

$$\hat{r} \wedge (\vec{a} \wedge \hat{r}) = \hat{r} \wedge \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ \cos\theta & \sin\theta\cos\varphi & \sin\theta\sin\varphi \end{vmatrix}$$

$$= \hat{r} \wedge \begin{pmatrix} a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi \\ a_z \cos\theta - a_x \sin\theta\sin\varphi \\ a_x \sin\theta\cos\varphi - a_y \cos\theta \end{pmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta\cos\varphi & \sin\theta\sin\varphi \\ a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi & a_z \cos\theta - a_x \sin\theta\sin\varphi & a_x \sin\theta\cos\varphi - a_y \cos\varphi \end{vmatrix}$$

$$= \begin{pmatrix} \sin\theta\cos\varphi\,(a_x \sin\theta\cos\varphi - a_y \cos) - \sin\theta\sin\varphi\,(a_z \cos\theta - a_x \sin\theta\sin\varphi) \\ \sin\theta\sin\varphi\,(a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi) - \cos\theta\,(a_x \sin\theta\cos\varphi - a_y \cos\varphi) \\ \cos\theta\,(a_z \cos\theta - a_x \sin\theta\sin\varphi) - \sin\theta\cos\varphi\,(a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi) \end{pmatrix}$$

So integrating each component

$$\vec{F} = \frac{\mu_0 q^2}{16\pi^2 r_0} \begin{pmatrix} \int_0^{2\pi}\int_0^{\pi} (\sin\theta\cos\varphi\,(a_x \sin\theta\cos\varphi - a_y \cos) - \sin\theta\sin\varphi\,(a_z \cos\theta - a_x \sin\theta\sin\varphi))\sin\theta\; d\theta\; d\phi \\ \int_0^{2\pi}\int_0^{\pi} (\sin\theta\sin\varphi\,(a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi) - \cos\theta\,(a_x \sin\theta\cos\varphi - a_y \cos\varphi))\sin\theta\; d\theta\; d\phi \\ \int_0^{2\pi}\int_0^{\pi} (\cos\theta\,(a_z \cos\theta - a_x \sin\theta\sin\varphi) - \sin\theta\cos\varphi\,(a_y \sin\theta\sin\varphi - a_z \sin\theta\cos\varphi))\sin\theta\; d\theta\; d\phi \end{pmatrix}$$

$$= \frac{\mu_0 q^2}{16\pi^2 r_0} \begin{pmatrix} \frac{8}{3} a_x \\ \frac{8}{3} a_y \\ \frac{8}{3} a_z \end{pmatrix}$$

$$= \frac{\mu_0 q^2}{6\pi r_0} \vec{a}$$

Defining a quantity $m = \dfrac{\mu_0 q^2}{6\pi r_0}$ we have **Newton's second law:** $\boldsymbol{\vec{F} = m\vec{a}}$

If $\vec{F} = 0$ then $\vec{a} = 0$ so **A body will remain in a state of uniform motion unless acted on by a force.** which is **Newton's first law**

Newton's third law is really a statement of the obvious. The force $\vec{F} = m\vec{a}$ is doing work or adsorbing energy to change the kinetic energy of the mass. All non-magnetic forces are ultimately electrostatic forces equal to the sum of the forces between electrons and protons which come in equal and opposite pairs. The magnetic force *Bev* on an electron moving through a magnetic field appears to contradict this law, but Classical EM theory shows that the magnetic field has momentum and that if we take into account the effect of the change in direction of the electron's path on its contribution to the form of the magnetic field, Newton's third law is obeyed. Thus we have derived Newton's laws.