Faraday's Law

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Faraday's law describes how a change in magnetic flux Φ threading a circuit of N turns of wire induces a an *emf* or voltage V seeking to oppose the change in flux.

$$V = -N \frac{d\Phi}{dt}$$

It is not a fundamental law, but a law relating to the properties of a manufactured object.

Classical physics explains the voltage generated in a circuit through the movement of magnetic flux regarding it as the integral form of one of Maxwell's laws. We believe this to be wrong. In our unified theory, the motion of magnetic flux of flux density $\vec{B}$ through the background of the coexisting electric fields of all elementary charge particles with velocity $\vec{v}$ generates an electric intensity $\vec{E}$ given by:

$$\vec{E} = \vec{v} \wedge \vec{B}$$

It is wrong apply this law to the situation where magnetic flux appears to be cutting conductors because in general, the magnetic flux and the circuit do not move through each other. Yes, quantum strands of magnetic flux pass thorough a conductor when there is no relative motion, but as soon as there is relative motion, currents will be induced and the conductor generates its own magnetic field. The conduction band electrons are themselves in motion generating magnetic fields which closely surround them and exclude the strands of flux from the background field. The net result is that the electric intensity $\vec{E} = \vec{v} \wedge \vec{B}$ does not exist within the conductor. What actually happens is a process of linking and breaking with each other as a quantum fluxoid loop of the magnetic field and a loop of the circuit's field break with themselves and rejoin with each other to form a single loop, then break again on the other side of the wire to form two separate loops with the result that the strand of magnetic flux now passes on the other side of the wire. So in fact, the wire is never exposed to the electric field $\vec{E} = \vec{v} \wedge \vec{B}$ because the $\vec{B}$ in question does not pass through the wire. [This is true in general, but there are exceptions. If we look at the field surrounding a conductor due to a current in it, the field extends inside the conductor and variations of current will cause a movement of magnetic flux within the conductor. However, as stated above, the conduction band electrons cannot feel this movement because they are surrounded by their own magnetic fields generated by their motion through the background.]

It needs to be stated that all this stuff about magnetic flux cutting circuits makes far more sense across the road in the Electrical and Electronic Engineering Department than it does in the traditional core university departments of Mathematics and Physics. Indeed modern physicists are supposed to follow Einstein's belief that magnetic fields do not really exist. The authors whole motivation in developing this unified theory is to try to bridge the gulf between the two camps and come to a common understanding of how the universe and our man made machines work. Both sides need to concede that the truth lies in no-mans-land. Magnetic flux is real stuff, but while it does not cut conductors, the mathematical artefact $\mu_0 \vec{H}_{field}$ does and the mathematics of summing the forces produced by all the energy transfers integrates to give the classical answer.

The other concept to grasp is the fact that conduction band electrons move every which way at great speed, but only drift relative to wires at speeds measured in millimetres per hour.

We have already proved that the motion through a magnetic field, which in the absence of the electron has a flux density $\vec{B}$, of an electron with velocity $\vec{v}$ relative to the field field results in force upon the electron given by:

$$\vec{F} = q\,\vec{v} \wedge \vec{B}$$

Now we know that in the real situation of relative motion between a conductor and a magnetic field, the effect of induced (Circuital and eddy) currents prevents the magnetic flux from passing through the conductor, so the force law is more correctly written:

$$\vec{F} = q\,\vec{v} \wedge \mu_0\,\vec{H}_{field}$$

Our task is to prove that the forces acting on the individual conduction band electrons sum over a length δl of wire to give a volts per metre field which will integrate over the length of the circuit to give the resulting voltage $V = -N\,\frac{d\Phi}{dt}$. In the following proof we will use the symbol $\vec{B}$ to represent the mathematical artefact $\vec{B} = \mu_0\,\vec{H}_{field}$ understanding that in the absence of the conductor, a real flux density $\vec{B} = \mu_0\,\vec{H}_{field}$ exists.

The force on an individual electron is $\vec{F}_i = q\,\vec{v}_i \wedge \vec{B}$. The force $\delta\vec{F}$ on the electrons within a length δl of the wire is:

$$\delta\vec{F} = \sum_i q\,\vec{v}_i \wedge \vec{B} = -\vec{B} \wedge \sum_i q\,\vec{v}_i$$

We must express the velocity $\vec{v}_i$ of an electron relative to the magnetic flux as the sum of its velocity $\vec{w}_i$ relative to the wire and the velocity $\vec{u}$ of the wire relative to the magnetic flux.

$$= -\vec{B} \wedge \sum_i q\,(\vec{w}_i + \vec{u}) = -\vec{B} \wedge q\left(\sum_i \vec{w}_i + \sum_i \vec{u}\right)$$

But the sum of the velocities of the electrons relative to the wire is $n\,\vec{u}_d$: the number n of electrons times the drift velocity $\vec{u}_d$. So the total force on the electrons becomes:

$$\delta\vec{F} = -\vec{B} \wedge n\,q\,(\vec{u}_d + \vec{u})$$

$$\delta\vec{F} = \vec{B} \wedge I\,d\vec{l} - \vec{B} \wedge n\,q\,\vec{u}$$

The first term gives the force acting on an element $\delta\vec{l}$ of wire carrying a current I and the second term the effect which we equate to be the induced voltage per unit length. Therefore:

$$n\,q\,\frac{\delta V}{\delta l} = \vec{B} \wedge n\,q\,\vec{u} \cdot \delta\hat{l}$$

$$\delta V = \oint \vec{B} \wedge \vec{u} \cdot d\vec{l}$$

In a time δt the element of wire $\delta\vec{l}$ sweeps out an area δA such that:

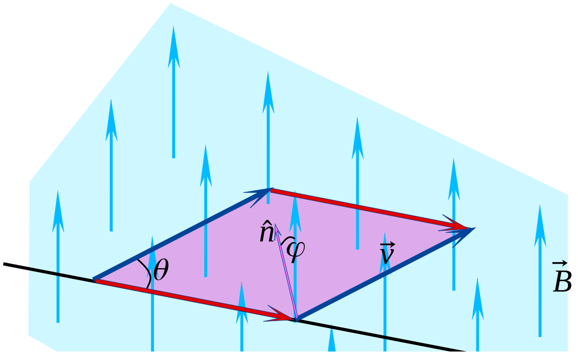
$$\delta A\,\hat{n} = \delta\vec{l} \wedge \vec{u}\,\delta t = \delta\vec{A}$$

where $\hat{n}$ is a unit vector perpendicular to the plane containing $\delta\vec{l}$ and $\vec{u}$. The flux $\delta\Phi$ threading this area is:

$$\delta\Phi = \vec{B} \cdot \delta\vec{A}$$

$$= \vec{B} \cdot \delta\vec{l} \wedge \vec{u}\,\delta t$$

$$\frac{\delta\Phi}{\delta t} = \vec{B} \cdot \delta\vec{l} \wedge \vec{u}$$



The dot product is commutative and the triple scalar product is invariant under cyclic rotation. Therefore:

$$\frac{d\,\Phi}{dt} = \vec{B} \wedge \vec{u} \cdot d\vec{l}$$

This equation now describes the end result rather than the process. So we may now regard Φ as being the real entity of the flux threading the circuit. Substituting, we get $V = -\frac{d\Phi}{dt}$. Faraday's is usually applied in situations where the circuit forms a coil of N turns of wire. The rate of change of flux $\frac{d\,\Phi}{dt}$ is for the whole coil. Since the direction in which the integration is taken is arbitrary, we determine the sign from energy considerations (as in Lenz's principle). So the induced voltage is:

$$V = -N\frac{d\Phi}{dt}$$

We have proved Faraday's law from our fundamental assertions