The Law of Biot-Savart

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The law of Biot-Savart:

$$\vec{B} = \mu_0 I \oint \frac{\hat{l} \wedge \hat{r}}{4\pi r^2} dl$$

allows us to calculate the magnetic field at a point in space due to a current I in a circuit.

We believe that there is a background against which the motion of electric charges generates a magnetic field and that this background simply consists of the electric fields of all elementary charged particles coexisting in space.

However, in the laboratory and in manufactured electromagnetic devices, we deal with currents moving through wires. The magnetic field generated by a current in a circuit is given by the Law of Biot-Savart. Here we develop a rigours proof of this law from our understanding of the nature of the magnetic interaction. It is important because we believe that the primary function of magnetic fields in nature is to store kinetic energy thus giving charged particles the property of inertial mass. The electromagnetic phenomena exploited in motors, generators, transformers and inductances are merely by-products.

SI units are used and the descriptors take their microscopic or vacuum form.
We make the following assumptions:

1 That there exists a background against which the electromagnetic interactions take place.

2 That an elementary charged particle of charge q_i and electric field $\vec{D}_i = \frac{q_i \hat{r}_i}{4\pi r_i^2}$ moving with velocity $\vec{v}_i$ relative to the background has an action:

$$\vec{H}_i = \vec{v}_i \wedge \vec{D}_i$$

3 That a magnetic field of magnetic intensity $\vec{H}$ and magnetic flux density $\vec{B}$ forms as a result of the sum of these actions:

$$\vec{H} = \sum_i \vec{H}_i \qquad : \qquad \vec{B} = \mu_0 \vec{H}$$

We regard $\vec{H}$ as mathematical artefact. It can be split into $\vec{H} = \vec{H}_a + \vec{H}_b + \vec{H}_c + \ldots$ where $\vec{H}_a \ldots$ are summations over various sets of elementary charged particles.

We regard $\vec{B}$ as a descriptor of the physical entity which we call magnetic flux, representing the property we call flux density. $\vec{B}$ is singular and we **may not** write $\vec{B} = \vec{B}_a + \vec{B}_b + \ldots$ It is however common practise to regard magnetic fields as belonging to particular objects when considering regions in which each is dominant. So that in deriving the law of Biot-Savart for a circuit, we neglect the effects of the earth's magnetic field and the VDU at the end of the laboratory without loss of rigor.

From assumption 2

$$\vec{H} = \sum_i \vec{v}_i \wedge \vec{D}_i \tag{1}$$

$$= \sum_i \vec{v}_i \wedge \frac{q_i \hat{r}_i}{4\pi r_i^2} \tag{2}$$

We consider the action of the elementary charged particles in a section $\delta\vec{l}$ of the conductor of a circuit.

$$\delta\vec{H} = \sum_{i \in C \cup L} \vec{v}_i \wedge \frac{q_i \hat{r}_i}{4 \pi r_i^2} \qquad (3)$$

where C is the set of conduction band electrons in δl and L the set of constituent parts of the lattice ions.

For regions far from the conductor compared to its diameter, the $\vec{r}_i$ are equal. Therefore

$$\delta\vec{H} = \left(\sum_{i \in C \cup L} q_i \vec{v}_i \right) \wedge \frac{\hat{r}}{4 \pi r^2} \qquad (4)$$

For an uncharged circuit, for every $q_i \in C$, there is subset $I \subset L$; such that $\sum_{j \in I} q_j = -q_i$; for a neighbouring lattice ion. If we now define $\vec{u}_i$ as the velocity of a conduction band electron relative to the conductor.

$$q_i \vec{u}_i = q_i \vec{v}_i - \sum_{j \in I} q_j \vec{v}_j \qquad (5)$$

Combining equations 4 and 5

$$\delta\vec{H} = \left(\sum_{i \in C} q_i \vec{u}_i \right) \wedge \frac{\hat{r}_i}{4 \pi r^2} \qquad (6)$$

While the $\vec{u}_i$ are directed in every direction, $w = \left| \frac{\sum_{i=1}^{n} \vec{u}_i}{n} \right|$ is the average drift speed of the conduction band electrons. The current is $I = A w \rho$ where A is the area of cross section and ρ the charge density of conduction band electrons. The number of conduction band electrons in δl is $n = \frac{\rho A \delta l}{q_i}$.

Therefore $\sum_{i \in C} q_i \vec{u}_i = \vec{I} \delta l$ and it follows that:

$$\delta\vec{H} = \vec{I} \delta l \wedge \frac{\hat{r}}{4 \pi r^2} \qquad (7)$$

We may now form an integral

$$\vec{H} = \oint \vec{I} \wedge \frac{\hat{r}}{4 \pi r^2} dl \qquad (8)$$

Which gives us the Law of Biot-Savart for the magnetic field due to a current through an isolated circuit.

$$\vec{B} = \mu_0 \oint \frac{\vec{I} \wedge \hat{r}}{4 \pi r^2} dl \qquad \text{or} \qquad \vec{B} = \mu_0 I \oint \frac{\hat{l} \wedge \hat{r}}{4 \pi r^2} dl$$

We have derived the law of Biot-Savart